Memory effect correlations in random scattering media over space, angle and time

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Challenge: controlling light deep within tissue

Light randomly scatters within tissue

Wavefront-shaping: "undo" scattering
Challenge: controlling light deep within tissue

Light randomly scatters within tissue

Wavefront-shaping: "undo" scattering

SLM

Target Plane
How do we form a focus deep within tissue?

Technique #1: Optical Phase Conjugation

Scattered wavefront

Light from "guidestar"

Phase conjugate wave

Light returns to guidestar location

DOPC recording

DOPC playback
Guidestar examples

1. Photodetector
2. Transducer
3. Ultrasound
4. TRAP/TRACK

This talk: *efficiently scanning* focused light deep within tissue.

**Goal:** want to scan focus around

**Equivalent:** maximize FOV of imaging with adaptive optics.
Talk Outline

1. The optical memory effect
2. The "shift/shift" memory effect
3. The generalized memory effect
4. Experimental demo of maximized scanning
5. Scanning further with time-gated light
The optical memory effect

- Well-known scattering correlation
- Speckle at a distance shifts around but does not change shape
The optical memory effect

- Well-known scattering correlation
- Speckle at a distance shifts around but does not change shape

Application: Imaging "through" thin scattering layers

The optical memory effect

- Original approach\(^1\) interested in *intensity-intensity* correlations:

\[
C_{I}^{ab|a'b'} = C_{I}^{(1)} + C_{I}^{(2)} + C_{I}^{(3)}
\]

"Memory effect"

\[
C_{I}^{(1)}(\Delta k) \propto (\Delta k L)^2 / \sinh^2(\Delta k L), \quad \Delta k_a = \Delta k_b
\]

\(^1\) S. Feng, C. Kane, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. 61, 834 (1988).
The optical memory effect

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"Memory effect"

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\]

- We will work with *field-field* correlations\(^2\), the square root of \(C_I^{(1)}\):

\[
C_I^{(1)}(k) = |\langle E(k)E^*(k) \rangle|^2 = |C(k)|^2
\]

\(^1\) S. Feng, C. Kane, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. 61, 834 (1988).

The optical memory effect

What does the memory effect look like within the transmission matrix?
The optical memory effect

Visualization of the optical memory effect possible in $k$ and $x$:

Scattering response to a point source: "Intensity propagator"

Banded structure in $T_x$
The optical memory effect: a simple derivation

Assume we know the average magnitude of transmission matrix:

\[ I(x_a, x_b) = \langle |T(x_a, x_b)|^2 \rangle \]

\[ \mathcal{F}_{2D}[I(x_a, x_b)] = \sum_{k_a, k_b} \langle T(k_a, k_b)T^*(k_a - \Delta k_a, k_b - \Delta k_b) \rangle \propto C(\Delta k_a, \Delta k_b) \]

Assume average intensity response to point source is shift-invariant:

\[ C(\Delta k_a - \Delta k_b) = C(\Delta k) \propto \mathcal{F}_{x_b \rightarrow \Delta k} [\langle I(x_b) \rangle] \]
The optical memory effect: a simple derivation

Recipe to measure the optical memory effect:

1. Put point source on input surface
2. Measure average intensity at output surface, \(<I(x_b)\>
3. Take Fourier transform to get \(C(\Delta k)\)

Intensity propagator \(<I(x_b)\> \rightarrow F_{1D} \rightarrow C(\Delta k) \rightarrow \text{Optical memory effect}\)
The shift/shift memory effect

What happens if we switch x's and k's?
The shift/shift memory effect

\[ T(x_a, x_b) \]

\[ F_{2D} \]

\[ k_b \]

\[ T(k_a, k_b) \]
The shift/shift memory effect: the Fourier dual

Wavevector response to a plane wave:
"k-space intensity propagator" $<\hat{I}(k_a,k_b)>$

$T(x_a, x_b)$

$F_{2D}$

$T(k_a, k_b)$
The shift/shift memory effect: the Fourier dual

- Identical derivation, x's and k's swapped
- Recipe to measure the shift/shift memory effect:
  1. Shine plane wave on input surface
  2. Measure average wavevector spread at output
  3. Take its Fourier transform to get spatial correlation $C(\Delta x)$
- Focus and scan within anisotropic material (e.g., tissue $g \sim 0.92-0.98$)
Experimental demo of shift/shift memory effect

Light from SLM (optical phase conjugation)

Experimental demo of shift/shift memory effect

Light from SLM (optical phase conjugation)

Green curve: focus intensity
Black curve: FT plane wave response

The tilt/tilt and shift/shift memory effects

Tilt/tilt correlation

\[ \delta(x_a) \rightarrow <l(x_b)> \]

Spatial impulse response

Scanning in \( k \)

Shift/shift correlation

\[ \delta(k_a) \rightarrow <\hat{l}(k_b)> \]

k-space impulse response

Scanning in \( x \)

How are these two effects connected?
The generalized memory effect: combining tilts and shifts

New input: "single ray"*

*Actually defined via the Wigner distribution, paper has math details:

The generalized memory effect: combining tilts and shifts

New input: "single ray"*

\[ \delta(x_a, k_a) \]

\[ (x_b^+, k_b^+) \]

\[ (x_b^0, k_b^0) \]

\[ <P(x_b, k_b)> \]

\[ (x_b^-, k_b^-) \]

Space-angle response \[ <P(x_b, k_b)> \]

\[ (x_b^+, k_b^+) \]

\[ (x_b^0, k_b^0) \]

\[ (x_b^-, k_b^-) \]

*Actually defined via the Wigner distribution, paper has math details:

The generalized memory effect: combining tilts and shifts

New input: "single ray"

Space-angle response $\langle P(x_b, k_b) \rangle$

2D Fourier transform of space-angle response gives tilt/shift correlation:

$$\mathcal{F}_{2D} [\langle P(x_b, k_b) \rangle] \propto C(\Delta k, \Delta x)$$

The generalized memory effect: combining tilts and shifts

New input: "single ray"

Space-angle response $\langle P(x_b, k_b) \rangle$

2D Fourier transform of space-angle response gives tilt/shift correlation:

$$\mathcal{F}_{2D} \left[ \langle P(x_b, k_b) \rangle \right] \propto C(\Delta k, \Delta x)$$

4D Fourier transform used when scattering is not tilt/shift invariant:

$$\mathcal{F}_{4D} \left[ \langle P(x_a, k_a, x_b, k_b) \rangle \right] \propto C(\Delta k_a, \Delta x_a, \Delta k_b, \Delta x_b)$$

The generalized memory effect: is it important?

Space-angle response $\langle P(x_b,k_b) \rangle$

Tilt/shift correlations $C(\Delta k, \Delta x)$

・ Tilting and shifting correlations generally not independent
The generalized memory effect: is it important?

Space-angle response $\langle P(x_b, k_b) \rangle$

Tilt/shift correlations $C(\Delta k, \Delta x)$

- Tilting and shifting correlations generally not independent
- Optimal tilt and shift combo can achieve larger scan range
Experimental setup

- Two experiments:
  1. Pencil beam response, $<P(x_b, k_b)>$
  2. Shift/tilt correlation function (shift both diffuser & sample)

- Tissue phantom samples (5 µm spheres in agar, $g=0.97$, 0.3 mm – 1 mm thick)
Average space-angle scattering response to pencil beam

0.3 mm thick

0.5 mm thick
Directly measured shift/tilt correlations

Direct measurement  FT$_{2D}$ of $<P(x_b,k_b)>$  Simple simulation

0.3 mm thick

0.5 mm thick
Directly measured shift/tilt correlations

Direct measurement  \( \text{FT}_{2D} \) of \( \langle P(x_b, k_b) \rangle \)  Simple simulation

0.3 mm thick

0.5 mm thick

\( \Delta k_b (\text{nm}^{-1}) \)

\( \Delta x_b (\text{nm}) \)
Directly measured shift/tilt correlations

Direct measurement  FT$_{2D}$ of $<P(x_b,k_b)>$  Simple simulation

0.3 mm thick

0.5 mm thick

Directly measured shift/tilt correlations

Direct measurement

FT$_{2D}$ of $<P(x_b,k_b)>$

Simple simulation

Shift-shift

Tilt/tilt[1]

Optimal shift/tilt

Scanning distances and the optimal rotation plane

<table>
<thead>
<tr>
<th>Memory effect</th>
<th>Adaptive Optics</th>
<th>Tilt plane</th>
<th>Scan range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift</td>
<td>Pupil</td>
<td>$-\infty$</td>
<td>$\sqrt{2\ell_{tr}/k_0^2 L}$</td>
</tr>
<tr>
<td>Tilt</td>
<td>Surface Conjugate</td>
<td>0</td>
<td>$\sqrt{6\ell_{tr}/k_0^2 L}$</td>
</tr>
<tr>
<td>Generalized</td>
<td>Sample Conjugate</td>
<td>$L/3$</td>
<td>$\sqrt{8\ell_{tr}/k_0^2 L}$</td>
</tr>
</tbody>
</table>

Optimally tilt and shift = Tilt around plane $L/3$ deep
Why is L/3 optimal? An intuitive picture

Stack of semi-random phase plates
Why is L/3 optimal? An intuitive picture

Stack of semi-random phase plates

Correct for top plate

distort scanned focus
Why is L/3 optimal? An intuitive picture

![Diagram of a stack of semi-random phase plates, showing distortion correction and correct for lower plate.](image)
Why is L/3 optimal? An intuitive picture

Stack of semi-random phase plates

Distort pattern

Distort focus
Why is $L/3$ optimal? An intuitive picture

Stack of semi-random phase plates

Correcting here also corrects for planes after focus and at edges
Why is L/3 optimal? An intuitive picture

Stack of semi-random phase plates

\[ \Delta k_a \]

Preferentially weight slightly higher correction plane
Towards a larger memory effect with time gating

- Goal: select early arriving "snake" photons for scanning
Towards a larger memory effect with time gating

- Hypothesis: early arriving snake photons offer larger scan range

\[
\langle \hat{I}(k_b) \rangle \rightarrow F_{1D} \rightarrow C(\Delta k)
\]

\[
\langle \hat{I}_b(k_b) \rangle \rightarrow F_{1D} \rightarrow C_b(\Delta k)
\]
Experimental setup

- Eventually obtained gating measurements in the spectral domain:

Ultrafast speckle evolution over space

- Time step per frame: 8.5 femtoseconds
- 360 µm thick tissue phantom

Ultrafast speckle evolution over wavevector

- Time step per frame: 8.5 femtoseconds
- 360 µm thick tissue phantom

Ultrafast speckle evolution over space and wavevector

Time gating extends the shift-shift memory effect 3-4X

To do: Put all of this together

1. Combine above: \( \mathcal{F} [\langle P(x_b, k_b, t_b, \omega_b) \rangle] \rightarrow C(\Delta k, \Delta x, \Delta \omega, \Delta t) \) ?
To do: Put all of this together

1. Combine above: $\mathcal{F}[\langle P(x_b,k_b, t_b, \omega_b) \rangle] \rightarrow C(\Delta k, \Delta x, \Delta \omega, \Delta t) \quad ?$

2. Implement with F-Sharp$^1$

---

$^1$L. N. Papadopoulos et al., "Scattering compensation by focus scanning holographic aberration probing (F-SHARP)," Nature Photon. 2016
To do: Put all of this together

Shift/tilt memory effect

\[ \delta(x_a,k_a) \]

\[ \langle P(x_b,k_b) \rangle \]

Time gating

\[ \delta(t) \]

\[ \langle Q(t) \rangle \]

1. Combine above:

\[ \mathcal{F} \left[ \langle P(x_b,k_b,t_b,\omega_b) \rangle \right] \rightarrow C(\Delta k, \Delta x, \Delta \omega, \Delta t) \]

2. Implement with F-Sharp\(^1\)

\[ \text{[Images of wavefronts and scattering compensation]} \]

\(^1\)N. Papadopoulos et al., "Scattering compensation by focus scanning holographic aberration probing (F-SHARP)," Nature Photon. 2016
Thank you!

Joint work with:

University of Twente:
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Ivo M. Vellekoop

Charité Medical School:
Yiannis Papadopoulos
Nick Kadobianskyi
Benjamin Judkewitz

2018: starting as an assistant professor in Duke University's Biomedical Engineering Department, contact me if you'd like to chat!
Quick derivation of L/3 depth:

Fokker-Plank model for correlation in anisotropic scatterer:

\[
C^{FP}_W(\Delta r_b, \Delta k_b) = \exp \left( -\frac{L^3 k_0^2}{2\ell_{tr}} \left[ \frac{|\Delta k_b|^2}{3k_0^2} - \frac{\Delta k_b \cdot \Delta r_b}{k_0 L} + \frac{|\Delta r_b|^2}{L^2} \right] \right). \tag{10}
\]

Find optimal \(\Delta k_b\): take derivative w.r.t. \(\Delta k_b\) and set to 0:

\[
\Delta k_b^{opt} = \frac{3k_0 \Delta r_b}{2L}.
\]

Use \(\Delta k_a = \Delta k_b\) and \(\Delta r_b = \Delta r_a + \Delta k_a k_0 / L\):

\[
\Delta k_a^{opt} = \frac{3k_0 \Delta r_b}{2L} \quad \text{and} \quad \Delta r_a^{opt} = -\Delta r_b / 2.
\]

\[
\tan(\theta) \sim \theta = \left( \frac{1}{k_0} \right) \times 3k_0 \Delta r_b / 2L = \Delta r_b / 2x
\]

\[
x = \Delta r_b / 2 \times 2L / 3 \Delta r_b = L / 3
\]
Time gating shift/shift correlations with physical shifting
Principle of F-Sharp

\[ |E_{PSF}(x)|^4 \ast \text{Object}(x) = \text{Image}(x) \]

F-SHARP microscopy

\[ |E_{PSF}(x)|^3 e^{-j\phi_{PSF}(x)} \ast E_{PSF}(x) = E_{recon}(x) \]

Evolution of corrected strong beam after every correction step

1st \rightarrow 2nd \rightarrow 3rd Correction step